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ABSTRACT

The English translations of the first six volumes of Studies on the Teaching and Learning of Mathematics in the Soviet Union are viewed in this document. The contents of each volume are briefly described: vol. 1, The Learning of Mathematical Concepts; vol. 2, The Structure of Mathematical Abilities; vol. 3, Problem Solving in Arithmetic and Algebra; vol. 4, Problem Solving in Geometry; vol. 5, The Development of Spatial Abilities; and vol. 6, Instruction in Problem Solving. General comments are made about the nature of research in the Soviet Union, and specific comments are made about individual sections within each volume. In a final section, some general remarks are included. First, the classroom orientation of the studies is noted, with the practicality and the paucity of statistics noted. Second, the choice of articles is questioned. Third, the amount of repetition is noted. Finally, it is concluded that these volumes can be an inspiring source of ideas. (MS)

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A review by

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of

Soviet Studies in the Psychology of Learning and Teaching Mathematics **(6 volumes), edited by Jeremy Kilpatrick and Izaak Wirszup**

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BOOK REVIEW*

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Soviet Studies in the Psychology of Learning and Teaching Mathematics (6 volumes), edited by Jeremy Kilpatrick and Izaak Wirszup. Chicago: School Mathematics Study Group, Stanford University, and Survey of Recent East European Mathematical Literature, The University of Chicago.

Vol. I The Learning of Mathematical Concepts. 1969. ix + 216 pp. $2.00.

Vol. II The Structure of Mathematical Abilities. 1969. ix + 128 pp. $2.00.

Vol. III Problem Solving in Arithmetic and Algebra. 1969. ix + 183 pp. $2.00.

Vol. IV Problem Solving in Geometry. 1970. ix + 154 pp. $2.00.

Vol. V The Development of Spatial Abilities. 1971. ix + 168 pp. $2.00.

Vol. VI Instruction in Problem Solving. 1972. ix + 136 pp. $2.00.

In 1953, H. E. Fehr was editor for the 21st yearbook of the National Council of Teachers of Mathematics; the

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book was entitled The Learning of Mathematics, Its Theory and Practice (Fehr, 1953). The book contains articles on a lot of topics relevant to teachers of mathematics: motivation, concept formation, language, training, excercises and training schemes, problem solving, individual differences, and so on. Most of the articles are reviews of literature on these themes. In the reviews the authors discuss the research and theory building of learning psychologists, try to find "implications for teaching," and form statements of advice for teachers. It is clear that the rationale is: Explore what is known about learning and try to use this knowledge in your teaching.

Twenty years later, in 1973, Crosswhite and others edited Teaching Mathematics: Psychological Foundations. (Crosswhite, Higgins, Osborne, & Shumway, 1973). In the opening article Shulman expresses optimism about the "growing psychology of learning that is finally becoming meaningful to curriculum construction and educational practice." But he adds: "It can never be immediately translatable into a psychology of the teaching of mathematics or science." And it is true. In the book, one finds articles about general topics (such as the theories of Gagné; the relation between mental growth and instruction; the theories of Piaget, of Bruner, of Ausubel, and of others; the controversies on discovery learning, etc.), but the reader himself has to look for the "implications for teaching." In this task he is helped a little by the questions and tasks at the end of each article.

In spite of the claimed relevance, we must say that we don't know of many examples of the real influence of learning psychology on the classroom practice of mathematics teachers and curriculum developers. What is the reason for this lack of influence? We think one reason is the lack of energy invested in the confrontation of learning theory and classroom practice. Indeed, it is a confrontation--not a translation, not an application, but a creative process in which a curriculum specialist or teacher scrutinizes the literature, thinking of the lessons or worksheets he has to prepare, and then tries to inspire himself, through an experimental design, a sentence, a conclusion, or an association. Working out an idea can take months of work!

But we are also sure that another reason for lack of influence is to be found in learning theory itself. Many of its topics are not interesting or inspiring for the

practitioner. And if the topic is interesting--as, for example, when it deals with concept attainment and concept formation--the restraints of the laboratory setting make the results either irrelevant for practice or trivial. (An example of this is to be found in the summary of 250 investigations made by Clark, 1971.) Of course we know that reliability is an important scientific principle, but so are validity and relevance.

In the Soviet Union, psychologists have left their laboratories for the schools, the training fields for sports, and so on. They are following a tradition of European psychologists such as Wertheimer, Koffka, Selz, Duncker, Kohnstam, Vygotsky, and others of the 1920s and 1930s. Common to all of them is the quest for the essentials of higher mental processes of cognition, emotion, and volition; and most of them thought their work was important to education and instruction.

The Soviet psychology of the last 30 years has known a fast-growing interest in education and instruction, not only visible in the choice of topics but also in the methods of research and the form of publications. This interest is ideologically rooted in the Marxist philosophy, as is indicated by Menchinskaya in her review of Fifty Years of Soviet Instructional Psychology (1967):

> Soviet instructional psychology concerns itself with the characteristics of mental activity during instruction and with the principles of the learning process itself. But since instruction is the main condition for a person's development during his lifetime, this branch of psychology has acquired considerable importance for disclosing general psychological principles. (Vol. I, p. 5)

Kilpatrick and Wirszup chose this review as the opening article for the first volume of their series of translations of Soviet Studies in the Psychology of Learning and Teaching Mathematics. Menchinskaya writes about the developments in Soviet psychology since 1917. In 1936 the Central Committee of the Communist Party condemned "pedology" and its belief in laws of maturation and heredity as the basis of individual differences. Since then the function of psychology would be "rationally

organizing the developmental process in school." There is not much attention to cross-sectional research but more to teaching experiments and longitudinal research in classes, with attempts to optimize teaching strategies and the choice of content in order to stimulate the development of the pupils (as to tempo and quality). Researchers try to get in hand the essentials of the learning processes by observing children at work, analyzing their mistakes, interviewing them on problem-solving methods, and asking them to "think aloud." This is an intensive way of working; probably it is the reason why in most cases the experimental groups are rather small (varying from 6 to 60 children). The research reports show attention to the details of instruction and learning; there are protocols of conversations between children and the teacher-researcher, drawings and photographs of worksheets, and so on. There is not much attention, however, to the type of research called "hypothesis testing." Questions of generalization, statistical reliability of assertions, and analysis of variance are neglected by most of the researchers; nevertheless, the choice of topics, as well as the narrow relationship to teaching in the classroom, and the ways of presentation make much of the work of Soviet psychologists a source of inspiration for teachers in mathematics and other subjects, as well as for curriculum specialists and psychologists of learning and instruction who would try to test assertions in more controlled settings.

The six volumes, edited by Kilpatrick and Wirszup, contain 36 articles by different authors. Each volume has an introduction in which the articles are summarized; in a very few cases there are some critical remarks or references to Western literature.

Volume I opens with the article by Menchinskaya mentioned above. Menchinskaya herself was and still is an active participant in the developments she describes. She has formed her own "school" in the USSR with its own research tradition and major concepts. One of the concepts that is very important is that of "teachability" or "receptivity to instruction." Teachability is the extent to which, in the pupil, definite qualities of mind can be formed. It is not an IQ, or something like that, but a concept that inspires a search for the frontier of the potentialities of children that receive good instruction. These frontiers can only be found in working with the children. A great deal of research is reported--

in this article and elsewhere--that indicates that children have broader potentialities than had ever been thought before!

But there are psychologists in the USSR who take other positions; therefore it is a good idea to read the introduction to A Handbook of Contemporary Soviet Psychology by Cole and Maltzmann (1969), who give a broader historical description of the developments in Russian psychology, and from different standpoints. In the articles, chosen by Kilpatrick and Wirszup, these "different positions" are found but always "hidden" in choices of topics, explanations, and references. Nowhere are the theoretical positions of different authors made explicit or discussed. We will return to this matter later. First we will try to give some impression of the articles chosen by Kilpatrick and Wirszup.

The rest of Volume I contains five articles on topics in the domain of learning mathematical concepts. A. R. Luriya writes "On the Pathology of Computational Operations." The article, written in 1946, describes his work with patients suffering from aphasia. The hypothesis is that the errors made in reading and writing numerals and in performing numerical operations reveal the structure of the mental processes underlying these actions. So, by analyzing these errors, one can find out how a system of knowledge, concepts, and operations atrophies. And--reasoning conversely--one can find the laws concerning the difficulties in the mastery of numbers and numerical operations. Luriya gives a careful analysis of the loss of mastery; it becomes clear that the concept of place-value is essential. It is not that the "direct" significance of the digit is lost, but that the "system significance" appears to be essential. A number is--psychologically seen--a wealth of potentialities. The number is embedded in a network of relations (also language relations) and operations. It is the structure of this network that is essential for computation--not the "simple" concept itself.

This same point is stressed in the other articles of this first volume. N. A. Menchinskaya writes about "The Psychology of Mastering Concepts: Fundamental Problems and Methods of Research" (1950). In her view, mastering concepts is not a spontaneous process, nor is it a kind of "inserting into the student's store of knowledge," but rather a process in which a child's history, his emotional

responses, his active intellectual responses, and his instruction are all important. Abstraction and perception are parallel: sometimes perception can hinder abstraction; sometimes labels and definitions can strongly stimulate the process of concept formation, but a definition can also come too early, as well. Important aspects of the process are the following.

1. Giving more than one definition of the concept.

2. Using the concept in different situations.

3. Being verbally conscious of essential properties, as compared with nonessential ones.

4. Embedding the concept in a system of concepts and relations (and make this embedding explicit).

It is clear that there is a close relation to the experimental results and theories of American psychologists such as Heidbreder, Hunt, Huttenlocher, and especially Ausubel.

Two articles by V. I. Zykova deal with the mastery of geometrical concepts and the use of these concepts in solving geometry problems. She has worked with sixth-grade pupils on problems of Euclidean geometry with parallel lines, bisectors, and angles. Whoever teaches this kind of geometry knows the problems: Children pay attention only to special features of concepts, they cannot re-interpret drawings, they have difficulties in seeing more than one concept in a drawing, they are hindered by unusual positions of drawings, and so on. Zykova interprets these problems--after describing them very clearly--as caused by the lack of mental activities in working with concepts and by the lack of structure in the presentation of the system of concepts and rules.

After this analysis (we can only roughly summarize the careful and detailed descriptions of Zykova!) she describes her teaching experiment, in which she tried to "construct" instruction that stressed broader concept formation. The ways in which drawings and problems are presented are very important, as are their relation to the verbalization by pupils and teacher. Much attention is paid to a careful planning of the details of this "language game." The descriptions can be inspiring for all mathematics teachers; analyzing, discussing, and

duplicating the work of Zykova in teacher-training courses can give very good insight into essential problems of learning and teaching mathematics. And the effects of these courses can certainly be enhanced if the descriptions and "little theories" of Zykova are brought into relation with the work of Gagné, Ausubel, Hunt, and other psychologists.

The conclusions of Zykova are not based on a pretest-posttest design but only on records of pupils' work and conversations. But we think that the analysis is done very well.

The last article of Volume I is written by P. Ya. Cal'perin and L. S. Georgiev and is devoted to "The Formation of Elementary Mathematical Notions." Gal'perin and Georgiev stress that a number is the result of a measurement. The essentials of this approach are listed.

1. A quantity (in the beginning especially continuous quantities such as lengths, volumes, and weights) is compared with another quantity; essential is the isolation of the dimension by which the children compare.

2. Gradually the necessity of a "unit" of measurement is made clear.

3. The size of the quantity is seen as a quotient: V(olume)/U(nit) = S(ize).

Note that in the curriculum of Davydov and Elkonin for the first years of the primary schools, the children use letters to write down this type of expression--algebra precedes arithmetic (children also work with the expressions by transforming them).

4. The unit itself can consist of parts. (From the beginning children work with fractions.)

5. The quantity is not presented as an entity; neither are the units, but the relation is stressed. Number is a relationship between a quantity and a unit.

6. The size of the unit does not matter; what is important is choosing a unit handily; what is also important is the relation between different units.

Gal'perin and Georgiev state that this approach can prevent a situation in which "the child's reasoning is determined by the spatial dimensions of the quantities, which visually predominate over the unit of measure" (193). They consider it an important goal that "the child can precisely distinguish the quantitative, descriptive approach to a thing and the scientific, mathematical approach to it" (195).

Psychologically speaking, this view has led Gal'perin to constructing a theory of the "formation of new, complete mental actions." This theory is not only a learning theory, but also a teaching theory. In his opinion, the teacher has to "construct" the new mental action in the child's mind by taking care that the child's learning does not take place spontaneously and by chance, but in a controlled manner. This means that the learner successively "walks through" the following stages.

1. Constructing a "base of orientation" in which the fundamental insights into the necessity of evolving a mental action and its functioning are developed. A bad base of orientation is built by trial and error (as in the traditional, visually dominated number concepts). A better base is constructed if the teacher takes care that the child is informed about the specific function of the mental action to be learned. But the best base is that the child himself form a more general base of orientation by scientifically analyzing the problems to be solved. To reach this third type of base the teacher has to take care that the child goes through a very carefully arranged set of situations, experiments, verbalizations and expositions. Gal'perin believes that--if this type of instruction is realized--the mental development of children can be strongly accelerated and widened. (There are evaluation experiments of Ponomarev, Puskin, Kalmykova, and others that seem to support this anti-Piagetian belief.)

2. The (mental) action is carried out at a material or materialized level (with objects or cards with pictures or words).

3. The (mental) action is carried out by speaking aloud; in the beginning, accompanying the material action; later, only aloud and independent of the material action.

4. The (mental) action is done in "internal speech" and with curtailments.

5. The mental action is totally internalized.

At each level the action must be controlled by the teacher.

It is clear that there are essential relations between this theory and the ideas of Vygotsky, Luriya, and Leontew, and with the marxist philosophy on the function of consciousness and the believed evolution of consciousness out of social practice.

In the article by Gal'perin and Geogiev, this theory (that we described very roughly) is mentioned briefly. We think it is a pity that the editors did not pay more attention to an analysis of the theoretical positions of Gal'perin and some of his collaborators like Davydov, Karpova, and Tjoplenkaja. There is a great deal of research based on this theory--especially in the field of mathematics and science--that so far is painfully missed in the volumes edited by Kilpatrick and Wirszup.

Volume II is entitled The Structure of Mathematical Abilities. It contains four articles by the same author, V. A. Krutetskii. Though he does not pay much attention to the theory of "ability" and "aptitudes" and questions of heredity, and with a view to the resolution of 1936 painfully avoids the discussion of individual differences in abilities and aptitudes, his research is very interesting. In the first report he describes his finding of three important aspects of mathematical ability from interviewing 62 experienced teachers of mathematics.

1. The ability to generalize mathematical material (concepts, knowledge, rules) of earlier problems and experience to solve new problems.

2. The ability to shorten reasoning processes; pupils with a good mathematical ability make faster and better curtailments.

3. The ability to switch from a direct to a reverse train of thought.

Krutetskii worked with 15 pupils of differing ability (as indicated by their teachers). He offered four groups of problems to the pupils: reasoning problems, proofs, and working with algebraic rules in transforming algebraic expressions in two directions, namely,

$$(a+b)^2 = \ldots \text{ up to } (4x + y^3 - a)^2 = \ldots \text{ etc.}$$

and,

$$a^2 - 2ab + b^2 = \ldots \text{ up to } b^2 + (x-a)^2 - 2b(x-a)$$

$$= \ldots \text{ etc.}$$

All the solutions, discussions, trials, guiding hints, and retrospective conversations of each pupil around each problem were recorded. (Krutetskii himself operated as the teacher, guide, and discussion partner; so in this way he obtained very rich material for analysis.)

It is worth noting that reading and discussion of the descriptions of Krutetskii's work can have the same rich function as indicated in our description of Zykova's work! In his analysis, Krutetskii is especially interested in the learning process that takes place during the weeks of the sessions. For, he claims, it is in this process that one can try to recognize the mathematical abilities and attitudes of pupils. This investigation is one of the best examples of the intensive, intelligent, and careful way in which the Soviet researchers try to study the learning of mathematics.

The results confirm the importance of the three selected abilities. Almost all the events in the learning processes can be understood in terms of less or more ability in one of the three. (Of course, this is not an example of a validation study by Western criteria. We must, however, state that this investigation of Krutetskii gives more insight into the essentials of mathematical activity and mathematical ability than a great deal of factorial analysis with respect to insufficiently defined concepts.)

The second investigation reports on Krutetskii's work over a period of three years with a very gifted 8-year-old girl, Sonya L. In the course of the three years, 279 problems were worked with the child (and sometimes the same problem was repeated after some months). All the conversations were recorded and analyzed, providing, again, a very rich body of material from which Krutetskii published some very interesting parts in this study. He summarizes the features of Sonya's solving processes and gives lots of examples. It is not possible to give a detailed description; we summarize the most important features.

1. Direct analytic-synthetic perception of the problem: (a) fast analysis of the mathematical structure, (b) fast evaluation of specific features of the problem, (c) fast identification of irrelevant information, and (d) fast recognition of the particular semantic mathematical structure of the particular problem.

2. Fast and correct curtailment of reasonings.

3. Fast generalization of mathematical material (Krutetskii speaks of "generalization on the spot" and in this expression he "hides" the "insight"; he thinks this term has only formal significance and can in no way serve as an explanatory concept.

4. Good ability to recall the complete reasonings and procedures, if the curtailments lead to difficulties or if complete reasonings and procedures are asked for.

5. Good flexibility, especially the ability to reverse "trains of thought".

6. A striving for economy of thought, clarity, simplicity, and economy of solutions.

7. Good memory, especially directed to generalized methods of solving typical problems, basic schemes of reasoning and proofs, and logical schemes.

8. Bad memory for specific data of problems.

9. Good ability to visualize problems in three-dimensional mental representation and in finding means of graphic or schematic drawing.

10. A strong tendency to structure reality mathematically, looking for logical structures (e.g., in remembering poems), correlations, and functions.

11. A strong perseverance, that is, much work in a short time, and with much interest.

Krutetskii continuously tries to formulate in Pavlovian terms--the theory of the two signal-systems that are correlated. He also suggests a physiological base for Sonya's special ability. We leave aside these more theoretical questions. It is enough to say that in spite of all their methodological weaknesses and the questionable choice of problems of traditional Russian education and dull school practice, instead of real problems, these two investigations may be "eye-openers" for teachers and researchers.

The last two articles are only summaries or attempts at popularizing the first two. Another choice--for example, of the work of Kuljutkin in the area of problem solving--could have been more fruitful.

Volume III is Problem Solving in Arithmetic and Algebra. This volume contains articles by Menchinskaya, Yaroshchuk, Gurova, and Doblaev. In fact, there is little that is new and interesting in these articles for readers who studied the two investigations of Krutetskii in the second volume. Menchinskaya gives (in 1946!) a description of the "Intellectual Activity in Solving Arithmetic Problems." She identifies the same activities as mentioned by Selz, Sand, Andrae, and others and as illustrated and mentioned by Krutetskii. The "indications for teachers" at the end of her article are not sensational either.

1. Teach the pupils to ask questions about the problem.

2. Teach the pupils to discriminate the question in the problem and the conditions of the problem.

3. Teach the pupils to reformulate the whole problem in one sentence.

4. Teach the pupils to pay attention to contradictions in question and conditions.

5. Teach the pupils to break down the problem in partial problems.

6. Teach the pupils to visualize the problem and the questions.

Yaroshchuk gave 20 pupils five pairs of problems (numerical vs. word problems). The scope of his investigation was to study ability to typify the problem, that is, to recognize a known type in a given problem. He describes levels of typification and relations between this typification and the ability to operate.

Attention is given only to model problems of certain types and certain mathematical structures. The "model problems" are in most cases far from being realistic. It is not surprising that Yaroshchuk succeeds in interpreting the work of his pupils in terms of the Pavlovian doctrine mentioned earlier. A quotation makes further comment unnecessary.

> Typification is nothing other than actualization of an association whose first element is recognizing the model features of the conditions of the problem, and whose second element is either naming the type or using the model method of solution. (Vol. III, p. 68)

(If the second element is "naming the type," a new association must be made with the fitting model method!)

Though Gurova too works only with these model problems, her investigation is more interesting because she investigates the extent to which the awareness of one's own mental operations helps children in solving model arithmetic problems. The first part of the investigation begins with observing children solving problems (she finds the same difficulties and characteristics as Krutetskii, Menchinskaya, and others). Then she confronts each child with five or six descriptions of solutions (or half solutions) written out on sheets of paper, and she asks which of these descriptions is the solution he has found for himself. Analysis of the answers shows that better pupils are more aware of their own solving process. This holds as well for the structure of that process as for the sequence of utterances, numbers, and operations. The second part of the investigation consists of a teaching experiment in which Gurova tries to prove that it is the awareness of their own mental operations that makes some pupils able to solve new problems and others not, or less easily. She gives a number of protocols that seem to support this

hypothesis. Reading the protocols can be instructive for teachers and researchers, but her affirmative conclusion cannot be verified by the reader.

The article of Doblaev is a report on "Thought Processes Involved in Setting up Equations." Like Yaroshchuk, Doblaev works within the Pavlovian school of thought: "All learning consists of the formation of temporary ties, and this is thought, thinking, knowledge" (Vol. III, p. 106). He analyzes what associations must be learned in solving equations at the two teaching standpoints he describes: (a) you shall always start by looking for the unknown, and (b) you shall always start by forming the basis-equation. Further on he analyzes diverse types of equations and the association chains to be learned. This part of the report contains a lot of detailed analysis of pupils' written work and of the mistakes that are made. The descriptions are interesting for experienced teachers, but the conclusions are too readily drawn in Pavlovian terms.

In this third volume the investigations suffer from too close a connection with a type of mathematics instruction that has little to do with real mathematics and that is far from an educational philosophy that stresses comprehension, insight, individual development, independence, self-reliance, and responsibility for one's own learning.

Moreover, this work, and especially Yaroshchuk's and Doblaev's, shows a dogmatism and narrow-mindedness in the explanation of results that has nothing to do with real psychology. Even if it is true that Krutetskii also tends to interpret his findings in Pavlovian terms, he does so after his work with children and after exposing the results of his interested and scholarly observations of these children at work.

Volumes IV-VI contain research of a much less fundamental character than Volumes I-III. As far as the concern for teaching mathematics goes, most of the investigations published in Volumes IV-VI are strongly conditioned by the instructional system prevailing in the Soviet Union during the period 1945-1960, in which almost all of these papers were written. All of them appeared before B. V. Gnedenko (1962) and A. I. Markušievič (1963) resumed A. Ja. Hinčin's severe criticism of actual mathematical instruction. They also precede N. A. Menčinskaja and M. I. Moro's book (1965) which seems to have been

quite influential. As a matter of fact, even older work of N. A. Menčinskaja (1955) is rarely cited in these volumes. Though there is not much information about the degree to which Soviet mathematics instruction has actually been affected by more recent reforms, it may be taken for granted that the instruction described and analyzed in Volumes IV-VI stems from a bygone period. The decision of the editors to pay attention to this period can be fully understood within the framework of the whole edition. Though they did not say so, it may be assumed that the editors aspired to a paradigmatic completeness, which is indeed indispensable in order to understand more recent critical and constructive developments. Since we are not familiar with the older literature, we cannot judge the choice the editors made, but if inferences from the recent criticism are allowed, we would guess that the chosen literature is fairly representative.

The attitude displayed by the authors of almost all the papers is pronouncedly conservative as far as mathematics instruction is concerned. Its letter and spirit is expressed by a few widely used textbooks and a traditional methodology, accepted as an invariable datum and almost never questioned. The question of whether the subject matter taught is mathematically sound, didactically feasible, and worth teaching and learning is almost never raised. Most of the contributions are critical and careful, but never does the criticism turn fundamental, and the great care bestowed is directed to improvements in a given subject matter and a traditional teaching method. In fact, this attitude does not differ much from that displayed in other countries by researchers on mathematics teaching up to the late 1950s. It would be an anachronism to expect reform ideas of the 1960s in this collection.

Mathematics as understood in these volumes means arithmetic with a heavy stress on word problems such as was cultivated and brought to great perfection in Soviet mathematical instruction, traditional geometry, and some algebra. Problem solving is understood in most of Volumes IV-VI not as a free activity, as it is often nowadays, but as an activity of recognition and application of a narrowly restricted set of patterns. A summary discussion of the articles in Volumes IV-VI follows, in the order in which they have been published here.

E. N. Kabanova-Meller, "The role of the diagram in the application of geometric theorems" (1950, Vol. IV, pp. 7-49).

> First series of experiments. . . . In this series of experiments, our goal was to determine, first, what processes are involved in the correct application of theorems to simple problems and why some pupils are unable to use theorems they have studied; and second, how a pupil perceives a given diagram and what role is played, in the application of theorems, by visual images--either images from memory (i.e., mentally reproduced diagrams from the book), or the representation of spatial figures according to a given diagram. (p. 9)

The experiments were based on assignments that involved the application of the two following theorems of solid geometry.

> Theorem: If two intersecting lines (AB and AC) in one plane (P) are respectively parallel to two lines (A_1B_1 and A_1C_1) in another plane (Q), then the planes are parallel.

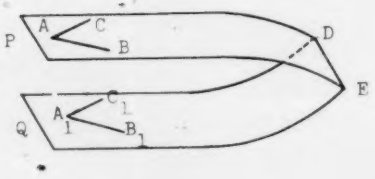

Figure 1

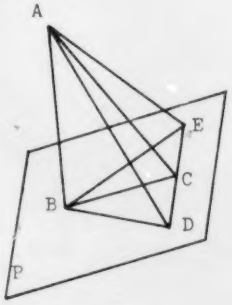

Figure 2

> Theorem: If a line (DE), drawn in a plane (P) through the base of an oblique line (AC), is perpendicular to the projection of the oblique line (BC), then it is perpendicular to the oblique line itself. (p. 10)

"The first task requires an application of the theorem of parallel planes: The lines drawn in planes 1, 2 and 3 (Figure 3) are respectively parallel. Are the given conditions sufficient to establish the parallelism of planes 1 and 2, 2 and 3, and 1 and 3?" (p. 11).

"The second requires an application of the theorem of three perpendiculars: The drawing (Figure 4) represents a cube. Mentally join vertex C to points A and B, and in triangle ABC determine the magnitude of the angle at vertex B" (p. 11).

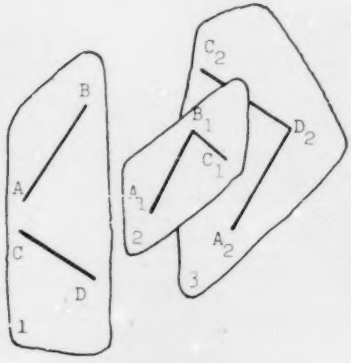

Figure 3

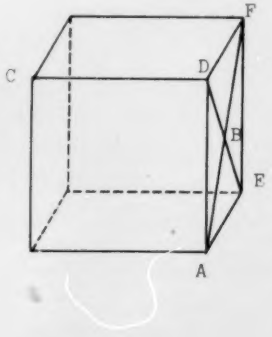

Figure 4

Both theorems and the diagrams, Figures 1 and 2, were taken from a widely used traditional textbook. We are not acquainted with this textbook, but if one is allowed to draw a conclusion from the examples, quoted here and at other places, it must be an unusually bad textbook. It is not easy to accumulate so many didactical mistakes within so little space as is exhibited here. Though the experimenter does not say so, her experiments do not reveal anything else but how dangerously pupils can be blocked by bad mathematics teaching and how this can be redressed by remedial teaching. In this respect the

investigation is valuable. If, however, criticism within this instructional system is allowed to be superseded by criticism from outside, the reproach of being blocked extends to the author of the textbook as well as to all of its users. In Figure 4, no reasonable person would prove the proposition by an appeal to the theorem of Figure 2, or even consider it in the context of solid geometry. It is mere plane geometry, and one needs years of training in solid geometry and in pedantry to overlook or disregard the isosceles triangle CAF, from which the proposition follows without any reasoning.

In the second series of experiments the questions are: "(1) What generalizations are necessary for correctly applying the plan of the proof to a diagram that is changed from the textbook diagram? (2) What role does the diagram used in learning the theorem play in applying the plan of the proof?" (p. 25).

The assignments involved two theorems and diagrams from the same textbook as that in the first series (pp. 25-26).

Theorem 44: An exterior angle of a triangle is greater than either of the interior angles that are not adjacent to the exterior angle.

Given: Triangle ABC and exterior angle BCD.

Prove: Angle BCD > angle ABC.

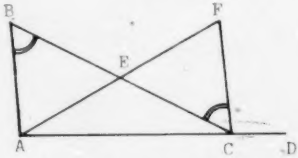

Figure 7

Through the midpoint E of side BC, construct median AE, and on its extension lay off segment EF = AE.

Join F and C by a line. Triangles ABE and EFC are equal [i.e., congruent (Ed.)], since they have equal angles at point E, which is located between two respectively equal sides. Hence angles B and ECF, lying opposite equal sides AE and EF, are equal. But angle ECF is part of the exterior angle BCD and therefore is smaller than the latter; hence angle B is also less than angle BCD.

Theorem 52: If two sides of one triangle are respectively equal to two sides of another triangle, then the greater side is opposite the larger of the angles contained between them.

Given: In triangles ABC and $A_1B_1C_1$: $AC = A_1C_1$, $AB = A_1B_1$, and angle A > angle A_1.

Prove: $BC > B_1C_1$.

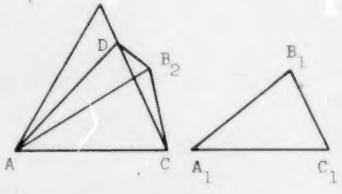

Figure 8

Superimpose triangle $A_1B_1C_1$ on triangle ABC so that side A_1C_1 coincides with AC. Since angle A_1 < angle BAC, side A_1B_1 will pass within the angle. Construct bisector AD of angle BAB_2 and join D with B_2: we obtain two triangles ABD and DAB which are equal (AD is a side common to both, $AB = AB_2$ according to the condition, and angle BAD = angle DAB_2 by construction). Since the triangles are equal it follows that $BD = DB_2$. Now from triangle DCB_2 we conclude that $B_2C < B_2D + DC$; hence $B_2C < BD + DC$. This means that $B_1C_1 < BC$.

Twenty-four sixth graders who had studied these theorems in class were asked to recall them and a few days later to reproduce the diagrams and the proofs. Afterwards the pupils were given changed diagrams and asked to prove the theorems while using the diagrams. The correct or incorrect reactions of the pupils were carefully observed and analyzed.

The proofs of both theorems show gaps, which are inherent in traditional geometry teaching. It would be hardly possible to fill them without profoundly scrutinizing the role played by diagrams in geometry teaching. Of course this cannot be done with sixth graders, but at least at the level of the textbook author this should be compulsory. Clearly, this has been neglected. It is not even clear whether the gross mistakes in the diagrams are intentional.

But our main objection concerns the bad teaching witnessed by the textbook and accepted as an invariable fact by the experimenter. Both proofs have had a long history. They are the result of long developments, which started with natural attempts, continued along a sequence of sterilizing simplifications, and ended with antiseptic solutions that, such as they are presented, can only be memorized but hardly understood. It is quite clear that even the textbook author did not grasp the origin of the unmotivated auxiliary lines. Fresh thinking would lead to natural proofs of the two theorems, but this would explode the narrow frame of traditional geometry teaching. In the proof of Theorem 44, the key problem is to memorize which

of the two medians should be drawn. After a few failures one will discover that it is just "the wrong one" that is to be chosen. But drawing both medians would lead to full understanding and to the source of the proof. Theorem 52 is easily understood as soon as circles are admitted, but unfortunately in a traditional geometry course they are not allowed so early.

The experimenter has focused on one variable, the diagram. The method of teaching was beyond the reach of the investigation.

N. F. Talyzina, "Properties of Deductions in Solving Geometry Problems" (1957, Vol. IV, pp. 51-101).

This paper is the only larger one in Volumes IV-VI that depends to a lesser degree on the particularities of the instructional system. It is also more psychological than didactical. Problem solving as defined by the author means applying general propositions to concrete situations, and the process by which this is done is called deduction--a strange terminology. A deduction may be complete or partial according to whether the subjects have total or partial recall of the definition or proposition they apply. The statements themselves may be objective--that is, about the properties of some diagram--or operational--that is, serving a proof or a construction. Whether the deduction is complete or partial is ascertained by interview--it is not clear whether the probing of the subject was profound enough to allow this distinction. The material used was quite simple geometry. The subjects were sixth through tenth graders. Partial deduction appears to be the rule, particularly in older pupils.

N. S. Mansurov, "The Influence of the Diagram and the Verbal Formulation of a Problem on the Choice of Solution Method" (1958, Vol. IV, pp. 103-107).

The paper is rather sketchy--a few examples that have little in common--and partly incomprehensible.

I. S. Yakimanskaya, "Some Features of Mental Activity Revealed in Reading a Diagram" (1958, Vol. IV, pp. 109-117) and "Individual Differences in Solving Geometry Problems on Proof" (1959, Vol. IV, pp. 119-125).

The greater part of these two papers is psychological research on the ability of children to analyze complex diagrams; more precisely, to discover closed graphs, or graphs of a given type within a larger geometrical diagram. There is no indication how this ability should be related to geometrical thinking, and the examples are of a kind that seems to exclude such a relation. The second paper also reports an investigation of the ability to reestablish the proof of the same theorem with diagrams in varying positions, but part of the reproduced diagrams are wrong and the text is too sketchy.

K. A. Slavskaya, "The Role of Reformulation in Problem Solving" (1959, Vol. IV, pp. 127-131) and "An Exploration Study of the Role of Reformulation in Problem Solving" (1960, Vol. IV, pp 133-135).

The experimenter investigated the reformulating behavior of eighth- and ninth-grade pupils who had to prove that the median of a trapezoid bisects every line segment joining the upper and lower bases. The theorem is a characteristic specimen of pedantry in traditional school geometry. The correct formulation would be, "Is there a line midway between two parallel lines and if so, try to find it," and the way to solve it would be to prove: If a line bisects <u>two</u> line segments joining the bases in <u>different</u> points, it bisects <u>all</u> such line segments. By the given formulation the learner is artificially l cked within an entirely irrelevant trapezoid. Neither the experimenter nor the subjects (except perhaps one of them) were able to break this blockade, but at this point the report is so confusing that we are inclined to conclude that either the experimenter or the translator did not understand what the subject meant to say.

L. L. Gurova, "The Influence of a Visual Aid on the Process of Solving Spatial Problems" (1960, Vol. IV, pp. 137-142).

Pupils solved construction problems on sections of solids without a concrete model, with the aid of a concrete model that left essential parts invisible, with a wire model, and with a glass model. The gains were observed. The report is sketchy; the shorthand descriptions of the solving strategies remain unintelligible even after attempts at correcting mistakes in the diagram.

E. N. Kabanova-Meller, "Positive and Negative Abstraction in the Process of Solving Geometry Problems" (1960, Vol. IV, pp. 143-147).

Abstraction here means classifying objects (e.g., geometrical shapes) according to one criterion (e.g., being a triangle). The degree to which, in the process of abstraction, the subjects still notice other differences (e.g., size, colour, or random features) is investigated. Abstraction can be isolating, accentuating, or dissecting. According to the author these different observation abilities should be trained.

E. N. Ivanitsyna, "Achieving Skill in Solving Geometry Problems" (1962, Vol. IV, pp. 149-153).

Eighteen pupils--no age indicated--were asked to prove that in a trapezoid the triangles formed by the diagonals and the lateral sides have the same area. Thirteen of them solved it. Their achievements are listed; in particular, all the unnecessary steps they made. It is a most revealing list.

N. F. Chetverukhin, "An Experimental Investigation of Pupils' Spatial Concepts and Spatial Imagination" (1949, Vol. V, pp. 5-56).

This is the most interesting contribution in Volumes IV-VI. The investigation starts with first graders and continues with fourth through sixth graders. Their spa-

tial imagination is tested by assignments to draw certain simple nongeometrical objects. The spatial imagination of eighth- to tenth-grade pupils and of college students is tested by geometry assignments of a quite unorthodox character, at least if compared with the concept of geometry prevailing in almost all other contributions.

Experiment 1

The following text (without a drawing) was to be dictated to the students, who were to be seated in a classroom or lecture hall (p. 23).

Imagine a cube whose upper face is designated by the letters ABCD, and whose lower face is designated by the letters $A_1B_1C_1D_1$, respectively. Thus, the vertex A_1 is under the vertex A, and so forth. In this cube a pyramid is constructed whose vertex is the point B, and whose base is the triangle $A_1B_1C_1$. In addition to this pyramid, imagine a second pyramid in the same cube whose vertex is the point B and whose base is the lower face $A_1B_1C_1D_1$ of the cube. Finally, imagine a third pyramid whose vertex is the midpoint S of the edge AB of the upper base of the cube, and whose base is the lower face $A_1B_1C_1D_1$ of the cube (Figure 24a). Thus you now have three pyramids: the first is $B(A_1B_1C_1)$, the second is $B(A_1B_1C_1D_1)$, and the third is $S(A_1B_1C_1D_1)$. Answer the

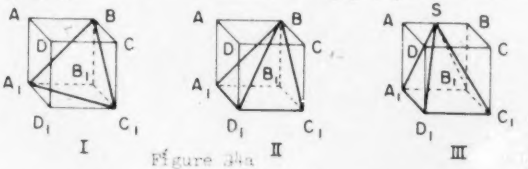

Figure 24a

following questions concerning each of the three pyramids mentioned above:

1. How many faces does the pyramid have?
2. Name the equal faces of the pyramid.
3. Name the equal dihedral angles of the pyramid.
4. Name the equal sides and equal angles in each face of the pyramid.
5. Name the right (plane and dihedral) angles of the pyramid.

Note 1:--The use of any drawing is prohibited; all answers should be presented in written form.

Note 2:--Plane angles are designated as usual by three letters. The dihedral angles of a pyramid are designated by the edge of the dihedral angle. For example, BB_1 is the dihedral angle formed by the faces AB_1B and CB_1B of the cube.

Experiment 2

On the board the instructor was to draw a cube in an arbitrary parallel projection (Figure 25). He was to inform the students that in the cube he had just represented on the board a sphere was inscribed, which he would not draw. Then he was to propose four sections of the figure (cube with an inscribed sphere):

1. A section through the midpoint of the lateral edge, parallel to the base.
2. A section at a height of a quarter of the lateral edge, parallel to the base.
3. A diagonal section.
4. A section through the midpoint of the edge of the upper base, parallel to the diagonal section.

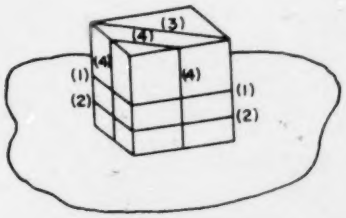

Figure 25

The sections were to be drawn on the board with the appropriate lines (Figure 25). The students were to be asked to construct the four sections in their true size and shape, given a segment with length equal to that of the edge of the cube (pp. 25-26).

Experiment 3

The pupils were given a definition and examples of a torus and were asked to draw its intersections with four planes orthogonal to the axis (Figure 27). (I did not reproduce the proper text, which consists of nonsense, probably due to the translator of this particular article, who in fact also made serious mistakes in other pieces.)

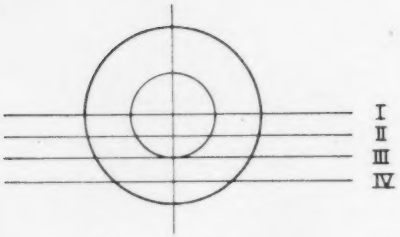

Figure 27

It is a pity that the results obtained with different groups of students are not strictly comparable, since the assignments have been subjected to considerable alterations. One conclusion, however, seems justified: College students of technical institutions who were trained in solid geometry performed relatively poorly compared to the younger students who had not been trained in this field. The author draws the conclusion that in teaching the geometry of space more attention should be given to strengthening spatial imagination.

G. A. Vladimirskii, "An Experiment Verification of a Method and System of Exercises for Developing Spatial Imagination" (1949, Vol. V, pp 57-117).

Though the author does not say so, his paper can be characterized as remedial teaching of spatial geometry in the wake of the same authoritative textbook as that followed by Kabanova-Meller. In the present paper the stress is laid upon developing spatial imagination, which seems to be completely disregarded in that textbook. The examples quoted in our report on Kabanova-Meller are resumed with the same lack of criticism; but the greater part of the remedial material is well adapted to its goal, though it would better serve as initial rather than remedial teaching.

O. P. Sergeevich, "The Formation of Children's Notions of Space in Connection with the Mastery of Elements of Geometry and Geography" (1956, Vol. V, pp. 119-144).

"The system of measures finds its visual expression in tables of measures"--to this statement quoted from authoritative didactic literature, and probably characterizing actual teaching of measures in many countries, the author is opposing his own view.

> Numerical expression of the correlation of metric measures is not a visual expression of the system of measures. On the contrary, these numerical relationships are an abstract expression of the visual and graphic correlation of specific quantities, of the real extent of the measures of length: the relationship of the length of one meter to one decimeter, to one centimeter, to one millimeter, etc. (p. 119)

Experiments with third graders showed that pupils were unacquainted with the real meaning of the various measures though they knew their mutual relations verbally; some of them did not even grasp what these relations meant in reality.

The chief aim of initial instruction in measures, according to the author, should be consolidation of the muscular and visual form of measures; whereas calculations of their quantity should be a secondary activity, subordinated to measurement. The author conducted interesting experiments with third graders while using the geographic map-making teaching unit, "The Plan."

I. S. Yakimanskaya, "The Development of Spatial Concepts and Their Role in the Mastery of Elementary Geometric Knowledge" (1962, Vol. V, pp. 145-168).

This plea for geometry instruction in the early grades is supported by evidence taken from L. Gurvich, "How I Taught My Son Geometry--First Lessons for Children" (Moscow, 1912). More recent experiments reported by the author are very traditional and have little to do with genuine geometry.

The majority of the contributions to Volume VI reflect a highly developed technique of teaching word problems according to fixed patterns. As mentioned earlier, this system came under fire in the late 1960s.

Z. I. Kalmykova, "A Psychological Analysis of the Formation of a Concept of Problem Type" (1947, Vol. VI, pp. 7-29).

The author strongly advocates teaching word problems according to types, for example, "equating" exemplified by

> Three pears and ten apples cost 4 rubles;
> one pear and one apple cost 75 kopeks.
> How much does each pear and each apple cost?
> (p. 11)

and "substituting" exemplified by

> Three pears and ten apples cost 4 rubles;
> one pear costs twice as much as one apple.
> How much does each pear and each apple cost?
> (p. 11)

Fourth graders were systematically taught clustering problems by types. Eleven different types were considered.

A. N. Bogolyubov, "Work with Words in the Solution of Arithmetic Problems in Elementary School" (1955, Vol. VI, pp. 31-60).

A large collection of practical advice on teaching word problems with attention to verbal analysis of the text, the influence of changes in verbal expression, and the translation of everyday language into mathematical language are the subjects of this article.

A. N. Bogolyubov, "A Combined Analytic and Synthetic Method of Solving Arithmetic Problems in Elementary School" (1955, Vol. VI, pp. 61-94).

For many years the method of teaching word problems was the embattled subject of adversary dogmatic theories, the so-called analytic and synthetic method, though as a matter of fact the definitions of "analytic" and "synthetic" showed considerable variations. According to one didactician "analytic" meant starting with the unknown elements and "synthetic" with the known elements; according to another the difference was starting with the text or with the numerical values; a third identified "analytic" with analyzing the text with respect to unknown and known elements and "synthetic" with reconstructing it from the data. Since such a dogmatic didactics is hardly feasible, it is not to be wondered at that the theoretic definitions were often contradicted by the examples that should serve to illustrate them.

The present paper is more pragmatic. Though the author calls his method a combination of analysis and synthesis, he simply recommends the most natural ways to solve word problems without prescribing a universal scheme. Of course, the fact of teaching this kind of word problem is in no way questioned.

A. E. Kozlova, "One Method of Analyzing the Condition of a Problem" (1955, Vol. VI, pp. 95-103).

The method, recommended as early as 1880, consists in asking the question "Why?" in order to disclose the basic relationships between the data in word problems. For example,

> Some textbooks were bought in a bookstore and were wrapped in two packages. The first package cost 36 rubles, and the second cost 40 rubles. There were 2 textbooks more in the second package. How many textbooks were in each package? (p. 99)

The question to be asked is "Why did one package cost more than the other?" or:

> First, 3 apples and 5 pears were bought at a total cost of 10 rubles 50 kopeks. Then, at the same price, 3 apples and 10 pears were bought, and 18 rubles were spent. How much did the apples and pears cost separately? (p. 100)

The question to be asked is "Why was the second more expensive than the first?"

M. E. Botsmanova, "The Forms of Pictorial Visual Aids in Instruction in Arithmetical Problem Solving" and "The Role of Graphic Analysis in Solving Arithmetic Problems" (1960, Vol. VI, pp. 105-110, 119-123).

Three kinds of pictorial aids are distinguished: first, "object-illustrative," that is, pictures of objects occurring in the problem that do not reflect the mathematical structure; second, "object-analytical," that is, pictures of such objects illustrating arithmetical relations; and third, abstract spatial diagrams and sketches, reflecting numerical relationships between the data. In textbooks the third kind is usually restricted to problems on motion, distance, and areas. The three kinds were investigated and compared. The reports on the results are sketchy.

G. P. Shchedrovitskii, "Toward the Analysis of Problem-solving Processes" (1962, Vol. VI, pp. 111-117).

The author has proposed a classification of solving processes expressed in a symbolic language. The brief representation is hardly convincing.

Yu. I. Gol'berg, "Methods Used by the Teacher in Problem-solving Instruction" (1962, Vol. VI, pp. 125-130).

The author criticizes the common classroom method of teaching problem solving where one pupil is active at the blackboard while the others note down the solution. He proposes systems where all pupils are active.

• A. V. Skripchenko, "Teaching Young Schoolchildren How to Solve Arithmetic Problems Through Comparison of Equations" (1963, Vol. VI, pp. 131-136).

This is the only plea in these volumes against the arithmetic methods of solving word problems and in favor of a moderately algebraic method.

Some General Remarks

1. As indicated, a general trait of the Soviet Studies edited by Kilpatrick and Wirszup is that almost all of them (except the study by Luriya) find their point of departure in the school practice of the Soviet Union. The same is true of the methodology of the studies: Almost all of them take place in classrooms or with groups of pupils in a simulated classroom-setting and with topics that are closely related to the school subjects. This close relationship with actual practice has some consequences.

a. The readability of most of the studies is good, because the problems are known to Western readers and also because most of the authors give detailed descriptions of the instruction, the reactions of pupils, and their mistakes. In short, there is a "smell of practice" in these studies, stimulating our reflection on our daily work as teachers or curriculum specialists.

b. Most of the studies neglect statistical matters. Questions of reliability, sampling, hypothesis testing, etc., are rare; answers, nearly absent. This is a drawback, but we must confess that a reliability-syndrome that leads to neglecting validity and relevance is even worse, in our opinion. In addition, in recent years statistical matters get more attention in the USSR than is suggested by these studies.

c. By closely relating research to actual practice, the Russians can throw light on the intrinsic problems of this practice and make suggestions for improvement "on the square meter." But never do we find a fundamental reflection on the goals and means of this practice! There are no

questions such as: (i) Is Euclidean geometry (Zykova, Talysina) a good means to teach children mathematics? (ii) Why should you teach children to solve model problems by typification? (iii) What is the real nature of the mathematical activity? To what extent and how can it be taught to children of different ages? What kind of mathematical problems and activities are worth choosing for instruction? What kind of instruction fits this philosophy of mathematical activity and mathematical education the best?

There are some "starters" to these questions, especially in the work of Krutetskii; also, in the work of Gal'perin. In each of the six volumes, however, we painfully miss a relationship between the educational questions of aims, or goals, and means. Here and there, especially in the articles of Menchinskaya, one finds some quotations of Marx and Lenin about the dialectic nature of human consciousness and the function of abstract thinking and mathematics. But our impression is that these are obligatory remarks; the questions are not really asked.

2. The last remarks bring us to some discussion about the editors' choice of articles. Nearly all of the articles are written by psychologists who base themselves on the work of Pavlov or on the elaboration of this work by Rubinstein, and, later, on work by Menchinskaya. But these are not the only Soviet "schools." In Volume I, for example, we find one article by Gal'perin. But this psychologist, and many of his students and collaborators, have done more work that in our opinion tackles the problems of learning and teaching mathematics at a more fundamental and sophisticated level than Menchinskaya and her students. We think it is a pity that this work is missing in the series--not because we agree in all aspects with the approach of Gal'perin, but because the work throws a new and sometimes inspiring light on fundamental problems.

The same is true of L. N. Landa and some of his collaborators who did a lot of work in investigating algorithms and the acquisition of algorithms--in grammar, in geometry, and in algebra. Also we could point to the work of Zankov and his students on discovery methods and to the work of Kuljutkin on problem solving. It is true that some of this work has been translated into German since 1969 (the year in which Kilpatrick and Wirszup

edited the six volumes), but a great deal of it was at their disposal before 1969 in German or in Russian.

For Western psychologists, we think that a closer look at the work of Joachim Lompscher in East Berlin would be valuable. He and his students have elaborated the ideas of Rubinstein in a system in which mental development and learning are joined in modes of analytical-synthetical operations. Lompscher takes fully into account the work of Piaget, though it is fundamentally criticized, and the work of Gal'perin, while basing himself on the ideology of Marxism-Leninism. In his work we see an attempt to connect diverse fields (ideology, a philosophy of education, a learning theory, a conception of mathematics) in a theory of instruction and learning. His work is an illustration of the statement that a learning theory or learning experiments can only be of help if one has an articulated idea of the nature of mathematical activity and of one's aims in mathematical education. Then a learning theory (even a Pavlovian one) can help in creating situations, in interpreting and guiding learning, and in formulating goals (or sometimes, if possible and desirable, objectives).

We think it is a pity that the editors did not discuss these burning questions and, so far, seem to have made choices that were too restrictive.

3. Repeatedly, we have indicated the function of some articles to inspire teachers and curriculum specialists to reflect on their work and to create new problems, new examples, and new explanations for the difficulties of children; but after reading the same stuff for the fifth or sixth time, inspiration loses its force! In our opinion, some articles would better have been skipped and been replaced by reviews in which the authors could have marked the differences between investigations and the results of different authors. This would have saved paper as well as reading time.

It is also a pity that there are almost no cross-references. Surprisingly, most of the authors do not seem familiar with the work of their colleagues let alone that of Western authors. (In recent years this acquaintance seems to have improved.) Even in the editorial preface the particular papers are only summarized and hardly related to, or compared to, each other.

We think that it would be a good thing if someone would make a thorough study to indicate "trends" in Soviet

and in Western literature. We would dare to predict that some certainty about some points would begin to grow and that this certainty could be based on research and theories from East and West. It is not too odd to see some relationship between Ausubel's plea for "anchor ideas" and "inclusive organizers" and Gal'perin's "base of orientation." Bruner and Olver's work on the role of perception in conservation and other concept attainments is related to the work of Zykova. Hunt and Hovland's theory of information processing in concept formation relates to the work of Huttenlocher, Kendler, Ausubel, and Landa.

4. Nevertheless, reading quite a few of the articles of these six volumes can be an inspiring activity for many categories of people working in the field of mathematics teaching.

References

Ausubel, D. P. Educational psychology--a cognitive view. New York: Holt, Rinehart & Winston, 1968.

Clark, D. C. Teaching concepts in the classroom: A set of teaching prescriptions derived from experimental research. Journal of Educational Psychology, 1971, 62(3), 253-278.

Cole, M., & Maltzmann, I. A handbook of contemporary Soviet psychology. New York: Basic Books, 1969.

Crosswhite, F. J., Higgins, J. L., Osborne, A. R., & Shumway, R. J. Teaching mathematics: Psychological foundations. Worthington, Ohio: Jones, 1973.

Fehr, H. E., (Ed.). The learning of mathematics, its theory and practice (21st Yearbook of the National Council of Teachers of Mathematics), Washington, D.C.: National Council of Teachers of Mathematics, 1953.

Gagné, R. M. The conditions of learning. London: Holt, Rinehart & Winston, 1965.

Gnedenko, B. V. Rol' matematiki i razvitii tehniki i proizvodstva. Matematika v Škole, 1962, 1, 32.

Heidbreder, J. & Huttenlocher, J. Some effects of negative instances on the formation of simple concepts. In R. C. Anderson, & D. P. Ausubel (Eds.), Readings in the psychology of cognition. New York: Holt, Rinehart & Winston, 1966.

Hunt, E. B. Concept learning: An information processing problem. New York: Wiley, 1966.

Markušievič, A. I. Ob očerednyh zadačah prepodavanija matematiki v škole. Matematika v Škole, 1963, 1, 11.

Menčinskaja, N. A. Psihologija obučenija arifmetika. Moskva: Učpedgiz, 1955.

Menčinskaja, N. A., & Moro, M. I. Voprosy metodiki i psihologii obučenija arifmetika v načal'nyn klassah. Moscow: Izd.Prosveščenie, 1965.

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